Math 871–872 Qualifying Exam
June 2011

Solve *three* problems from Section A and *three* more from Section B; you may work on any number of the problems, but indicate which six you want graded. When in doubt about the wording of a problem, ask for clarification. Do not interpret a problem in such a way that it becomes trivial. **Justify your answers**.

Section A:

(1) Prove that if $f : X \to Y$ is continuous map between topological spaces and C is a compact subset of X, then $f(C)$ is a compact subset of Y.

(2) Suppose $f : X \to Y$ is a quotient map. Prove that if Y is connected and $f^{-1}(\{y\})$ is a connected subspace of X for all $y \in Y$, then X is connected.

(3) Prove that every metrizable space is normal Hausdorff (aka T_4).

(4) Suppose A, B are disjoint, compact subspaces of the Hausdorff topological space X. Prove there are open subsets U, V of X such that $A \subseteq U$, $B \subseteq V$ and $U \cap V = \emptyset$.

Section B:

(5) Let X be the space obtained by deleting three distinct points from $\mathbb{R}^2$. Compute $\pi_1(X)$.

(6) View S^3 as the set of unit vectors in $\mathbb{R}^4$, and consider the equivalence relation on them induced by $u \sim v$ if $A \cdot u = v$, where A is the matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Compute the fundamental group of the quotient space $S^3/\sim$.

(7) (i) Describe a way of identifying pairs of faces of Δ^3 (the standard three simplex) to produce a Δ complex structure on S^3 having a single 3 simplex.

(ii) Write down the chain complex corresponding to the Δ-complex in (i). Be sure to include the differentials of the complex, but you do *not* need to compute the homology.

(8) Let X be the space obtained from the sphere S^2 by joining the North and South poles together with a straight line segment.

(i) Describe a structure of a CW complex on X.

(ii) Compute the homology of X using cellular homology, with decomposition from (i).