CHAPTER 7 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. For which of the following intervals does $P(x) = 4x^3 - 4x^2 - 33x + 45$ have a zero?

(A) $[-2, -1]$ (B) $[-1, 0]$ (C) $[0, 1]$ (D) $[2, 3]$ (E) $[3, 4]$

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2. Which of the following is the negation of the sentence $(S \vee T)$?

(A) $\neg S \wedge \neg T$ (B) $\neg S \wedge T$ (C) $S \wedge \neg T$ (D) $\neg S \vee \neg T$ (E) $S \wedge T$

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3. In a State Lottery, players choose 6 numbers from among the integers 1 through 51. The jackpot is awarded if the 6 numbers selected match the 6 numbers drawn. What's the minimum number of tickets someone would need to purchase in order to guarantee winning?

(A) 720 (B) 18,009,460 (C) 377,149,517
(D) 12,966,811,200 (E) 17,596,287,801

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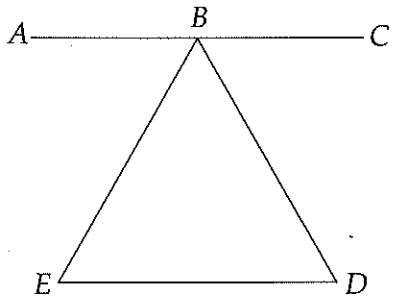
4. Using the undirected graph below, delete the edge DE. Which of the following must be true?



I. vertex D is even
II. the graph is a spanning tree of the previous graph
III. the graph is a forest

(A) None of the above (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

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5. A fair die is tossed twice. About how many times would you expect to roll 3 or greater?

(A) 2 (B) $\frac{3}{2}$ (C) 1 (D) $\frac{1}{2}$ (E) 0

---

6. Suppose A is the set of all prime numbers, and B is the set of all odd integers. Then the relative complement $A - B$ is equivalent to the set of

(A) $\{x \mid x \text{ is even}\}$
(B) $\{x \mid x \text{ is positive and odd}\}$
(C) $\{x \mid x \text{ is a composite integer}\}$
(D) $\{2\}$
(E) $\{0\}$

---

7. Which of the following sets is NOT countably infinite?

(A) $\mathbb{Q}$, the set of rational numbers
(B) $\mathbb{Z}$, the set of all integers
(C) $\mathbb{Q}^c$, the set of all irrational numbers
(D) $\mathbb{Z}^+$, the set of positive integers
(E) $\mathbf{E}$, the set of positive even integers

---

8. A man forgot the combination to his safe. The combination consists of a sequence of four numbers, each from 1 to 60. The man only remembers the following pieces of information:

All the numbers are different:
The second number is twice the third number.
The third number is prime.

How many possible combinations must the man try to unlock his safe?

(A) 1377 (B) 33,060 (C) 34,220 (D) 35,400 (E) 11,703,240

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9. What is the coefficient of the $(z-2)^{-1}$ term in the Laurent series for $f(z)=\dfrac{1}{z-5}$ centered at $z = 2$?

(A) 81 (B) 27 (C) 9 (D) 3 (E) 1

---

10. Let $g(x)$ be a polynomial function whose derivative is continuous and nonzero on the interval $[a, b]$. Suppose there exists a y on this same interval such that $g(y) = 0$. Let x_0 be an arbitrary x-value in the interval. Then x_1 is the x-intercept of the line tangent to $g(x)$ at x_0. For each subsequent n, x_n is the x-intercept of the line tangent to $g(x)$ at x_{n-1}. Which formula best approximates the root of $g(x)$ using the method described above?

(A) $x_{n+1} = x_n - \dfrac{g(x)}{g''(x)}$
(B) $x_{n+1} = x_n - \dfrac{g'(x)}{g''(x)}$
(C) $x_{n+1} = x_n + \dfrac{g(x)}{g'(x)}$
(D) $x_{n+1} = x_n - \dfrac{g(x)}{g'(x)}$
(E) $x_{n+1} = x_n - \dfrac{g'(x)}{g(x)}$

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11. The steps below are used to compute Euclid's Algorithm to find the greatest common divisor of two integers. If the numbers 380 and 72 are input, how many iterations of the algorithm will it take to find the gcd?

```
input a;
input b;
while (b > 0)  {
      int r == a mod b;
      a == b;
      b == r;
}
int gcd == a;
output gcd;
```

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

---

12. If $x^2 = 40$, use Newton's method twice to approximate the value of x to three decimal places.

(A) 6.223 (B) 6.225 (C) 6.320 (D) 6.323 (E) 6.325

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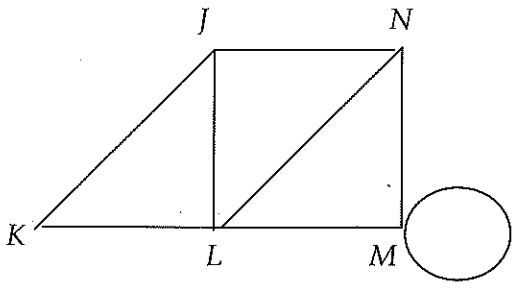
13. Given the undirected graph below, what is the maximum number of edges that can be removed that still leaves a connected subgraph?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

---

14. Find the Laurent series expansions of the function $f(z) = \dfrac{1}{z-3}$ that is valid in the annulus $|z-4| > 1$.

(A) $\sum_{n=1}^{\infty}(4-z)^{-n-1}$

(B) $\sum_{n=0}^{\infty}(-1)^n(z-4)^{-n}$

(C) $\sum_{n=1}^{\infty}(-1)^n(z-4)^{-n-1}$

(D) $\sum_{n=0}^{\infty}(-1)^n(z-4)^{-n-1}$

(E) $\sum_{n=1}^{\infty}(z-4)^{-n-1}$

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15. A teacher is assigning 6 students to one of three tasks. She will assign students in teams of at least one student, and all students will be assigned to teams. If each task will have exactly one team assigned to it, then which of the following are possible combinations of teams to tasks?

I. 90
II. 60
III. 45

(A) I only (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

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16. Of the 600 residents of Clermontville, 35 percent watch the television show *Island Survival*, 40 percent watch *Lovelost Lawyers*, and 50 percent watch *Medical Emergency*. If all residents watch at least one of these three shows, and 18 percent watch exactly 2 of these shows, then what is the probability that a resident chosen at random watches all of the shows?

(A) $\frac{1}{4}$ (B) $\frac{1}{5}$ (C) $\frac{9}{50}$ (D) $\frac{7}{100}$ (E) $\frac{7}{200}$

---

17. Which of the following is the solution set of the inequality $x+\frac{6}{x}>5$?

(A) $(0, 2) \cup (3,\infty)$
(B) $(0, 1) \cup (2,\infty)$
(C) $(-\infty, 2) \cup (3,\infty)$
(D) $(0, 2) \cap (3,\infty)$
(E) $(-\infty, 0)$

---

18. If the formula $A \vee B \rightarrow \neg C \vee D$ is true, which of the following statements is its contrapositive?

(A) $\neg C \wedge D \rightarrow A \wedge B$
(B) $C \wedge \neg D \rightarrow \neg A \wedge \neg B$
(C) $\neg C \wedge \neg D \rightarrow \neg A \wedge \neg B$
(D) $C \vee \neg D \rightarrow \neg A \vee \neg B$
(E) $A \vee B \rightarrow C \vee \neg D$

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19. Which of the following sets in $\mathbb{R}^2$ is closed?

(A) $(2, 5) \times (1, 3)$ (B) $(2, 5] \times [1, 3)$ (C) $[2, 5] \times (1, 3)$ (D) $(2, 5) \times [1, 3]$ (E) $[2, 5] \times [1, 3]$

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20. What is Log $(-e^3)$?

(A) -3 (B) 3 (C) $3+\pi i$ (D) $3-\pi i$ (E) $3+2\pi i$

---

21. Which of the following is NOT a tautology?

(A) $\neg(\neg A) \leftrightarrow A$

(B) $A \vee (\neg A)$

(C) $[(A \wedge B) \rightarrow C] \leftrightarrow [A \rightarrow (B \vee C)]$

(D) $\neg(C \wedge D) \leftrightarrow \neg C \vee \neg D$

(E) $A \vee (B \wedge C) \leftrightarrow (A \wedge B) \vee (A \wedge C)$

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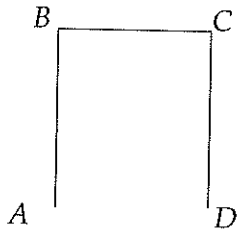
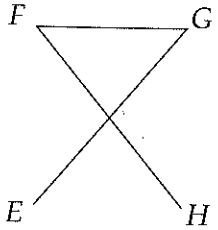
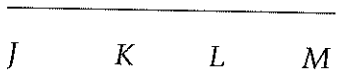
22. Which of the following graphs are isomorphic?



I



II



III

(A) None of the above (B) I and II only (C) I and III only

(D) II and III only (E) I, II, and III

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23. Given set $A = \{a, b\}$, set $B = \{c, d, e, f\}$, and set $C = \{g, h, i\}$. If set $D = P(C) \times B$, which of the following sets has the same cardinality as $P(D)$?

(A) $P[P(A) \times P(C)]$

(B) $P[A \times C]$

(C) $P(A) \times P(C)$

(D) $[P(B) \times P(C)]$

(E) $P[P(A) \times P(B)]$

---

24. What are the complex roots of the equation $e^{2z} = i$?

(A) $\left(\frac{i}{2}\right)\left(-\frac{\pi}{2} + 2n\pi\right)$

(B) $(2i)\left(-\frac{\pi}{2} + n\pi\right)$

(C) $\left(\frac{i}{2}\right)\left(\frac{\pi}{2} + n\pi\right)$

(D) $(2i)\left(\frac{\pi}{2} + 2n\pi\right)$

(E) $\left(\frac{i}{2}\right)\left(\frac{\pi}{2} + 2n\pi\right)$

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25. In the complex plane, the set of all points that satisfy the equation $(\bar{z})^2 = z^2$ is

(A) a circle (B) a point (C) a ray (D) a line (E) two lines

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26. Which of the following is a harmonic conjugate $u(x, y)$ of the harmonic function $v = x - 3x^{2y} + y^3$?

(A) $x^3 - 3xy^2 + y$ (B) $-x^3 + 3xy^2 - y$ (C) $-y^3 + 3x^{2y} - x$

(D) $y^3 - 3x^{2y} + x$ (E) $-x^3 + 3xy^2$

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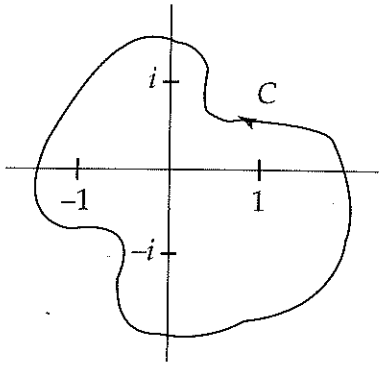
27. What's the value of the integral $\int_C \frac{z+1}{(z+3)(z^2+1)}dz$ where C is the curve shown below?



(A) $\frac{4}{5}\pi i$

(B) $\frac{2}{5}\pi i - \frac{1}{5}$

(C) $\frac{4}{5}\pi i - \frac{1}{5}$

(D) $\frac{2}{5}\pi i$

(E) $-\frac{2}{5}\pi i$

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28. Suppose X is a nonempty set for which $\delta : X \times X \to \mathbb{R}$ is a real-valued function defined on ordered pairs of points on X. Which of the following is NOT a property of the metric function δ?

(A) $\delta(x, y) \geq 0$

(B) $\delta(x, z) \leq \delta(x, y) + \delta(y, z)$

(C) $\delta(x, y) = \delta(y, x)$

(D) $\delta(x, y) = 0 \leftrightarrow x = y$

(E) $\delta(x, z) = \delta(x, y) + \delta(y, z)$

---

29. A baseball team consists of 20 players, 5 of whom are pitchers and 15 of whom are position players. If the batting order consists of 8 different position players and 1 pitcher, and if the pitcher always bats last in the order, then which of the following expressions gives the number of possible different batting orders for this baseball team?

(A) $\frac{15!(5)}{8!}$ (B) $\frac{15!(5)}{7!}$ (C) $\frac{15!(5!)}{7!}$ (D) $15!(5)$ (E) $20!$

---

30. What is the polar form of a complex number equal to $(i-\sqrt{3})^6$?

(A) -2^6 (B) $2^6(-1+i)$ (C) $2^6(1-i)$ (D) $2^6\left(\frac{1}{2}-\sqrt{\frac{3}{2}}i\right)2^6(\,)$ (E) 2^6

---

31. Which of the following sets in $\mathbb{R}^3$ is compact?

(A) $\{x, y, z \mid |x+y+z| < 5\}$

(B) $\{x, y, z \mid x < 2 \;\&\; y < 2 \;\&\; z < 2\}$

(C) $\{x, y, z \mid 0 \le x \le 3, 0 \le y \le 3, 0 \le z \le 3\}$

(D) $\{x, y, z \mid x \ge 0, y \ge 0, z \ge 0\}$

(E) $\{x, y, z \mid 2 < x^2 + y^2 + z^2 < 8\}$

---

32. If $f(x) = \begin{cases} \frac{x}{2} + c \text{ for } 0 \le t \le 8 \\ 0 \text{ otherwise} \end{cases}$, for what value of c is $f(x)$ the probability density function of a random variable X?

(A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) 0 (D) $-\frac{2}{3}$ (E) $-\frac{4}{3}$

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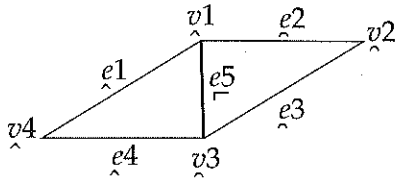
33. Which of the following represents the unoriented incidence matrix for the graph shown?



(A) $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$ (E) $\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

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34. Let (X, T) be a topological space, and let A be the subset $(0, 1) \cup [4, 6)$ in $\mathbb{R}$. Find the exterior of A.

(A) $(-\infty, 0) \cup (2, 3) \cup (6, \infty)$
(B) $[0, 1] \cup [4, 6]$
(C) $(-\infty, 0) \cup (1, 4) \cup (6,\infty)$
(D) $(-\infty, 0] \cup [1, 4) \cup [6,\infty)$
(E) $(-\infty, 0) \cup (6, \infty)$

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35. In how many ways can a company separate its 12 employees into 4 equally-sized committees?

(A) 369,600 (B) 184,800 (C) 61,600 (D) 384 (E) 48

---

36. Let C be the counterclockwise-oriented circle $|z| = 5$. If $h(a) = \int_C \frac{3z^2 + z - 4}{z - a}$, what is $h(3)$?

(A) $2\pi i$ (B) $16\pi i$ (C) $26\pi i$ (D) $52\pi i$ (E) $56\pi i$

---

37. If x is an element of the set $(A \cup B) \cap C$, which of the following must be true?

I. $x \in (A \cap B) \cup C$
II. $x \in (A \cup B)$
III. $x \in (A \cap B) \cap C$

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

---

38. Let X be a random variable on $\mathbb{Z}^+$ whose distribution function is $F_X(t) = \frac{1}{3^t}$. Suppose that W is another random variable whose distribution function is $F_Y(t) = \frac{1}{4^t}$. What is the probability that at least one of the variables X and Y is greater than 2?

(A) $\frac{5}{6}$ (B) $\frac{64}{81}$ (C) $\frac{1}{2}$ (D) $\frac{17}{81}$ (E) $\frac{1}{6}$

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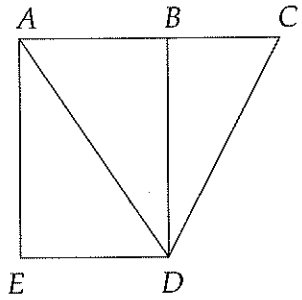
39. Which of the following sequences of vertices describes a circuit in the graph below?



(A) B, D, A, E, D, C, B
(B) C, A, E, D, C
(C) A, D, C, D, A
(D) E, A, D, B
(E) C, B, A, E, B, C

---

40. What is the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 2 & 3 & 1 & 4 \end{pmatrix}$ on the set $S = \{1, 2, 3, 4, 5, 6\}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

---

41. Let A, B, C, and D be well-formed sentences. Suppose $A \to C$, $B \to D$, $\neg C \to \neg D$, and $\neg A \to \neg C$. Which of the following statements must be true?

I. $A \to D$
II. A and C are logically equivalent.
III. $\neg A \to \neg B$

(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

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42. If $M = (-1, 3] \cup [7, 8)$, what is the Lebesgue measure of M?

(A) –5 (B) –1 (C) 0 (D) 5 (E) 9

---

43. A florist is arranging flowers in centerpieces at each of four tables at an event. If he has 61 flowers total, which of the following statements are true?

I One table has no more than 13 flowers.
II One table has at least 16 flowers.
III Two of the tables have at least 31 flowers combined.

(A) II only (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

---

44. A fair coin is flipped 10 times. What's the probability of getting between 40 and 50 "heads"?

(A) 10% (B) 38% (C) 41% (D) 47% (E) 53%

---

45. In how many different ways can four people sit around a restaurant table (not including rotations)?

(A) 24 (B) 18 (C) 6 (D) 48 (E) 16

---

46. For what complex number z does $\cos z = 3$?

(A) $i\log\left(3 \pm 2\sqrt{2i}\right) + 2k\pi$, for any $k \in \mathbb{Z}$
(B) $2k\pi - i\log\left(3 \pm 2\sqrt{2i}\right)$, for any $k \in \mathbb{Z}$
(C) $2k\pi - i\log\left(3 \pm 2\sqrt{2i}\right)$, for any $k \in \mathbb{Z}$
(D) $i\log\left(3 \pm 2\sqrt{2i}\right) + 2k\pi$, for any $k \in \mathbb{Z}$
(E) $i\log\left(3 \pm 2\sqrt{2i}\right) + 2k\pi$, for any $k \in \mathbb{Z}$

---

47. If $A \subseteq B$, which of the following conditions must be true?

I. $A \cup B = B$
II. $A - B = \varnothing$
III. $C - B \subseteq C - A$

(A) I only (B) I and II only (C) I and III only
(D) II and III only (E) I, II, and III

---

48. Let $f(z)$ be a complex analytic function such that $f(z) = (5x - 3y) + i\,v(x, y)$, where $v(x, y)$ is a real-valued function and $x, y, \in \mathbb{R}$. If $v(4, 1) = 7$, what is $v(3, 2)$?

(A) –10 (B) –9 (C) 1 (D) 9 (E) 14

Use the algorithm for the "3x + 1" problem for exercises 49–50.

```
input (a)
while a > 1
    begin
        if (a mod 2 = 0)
            a := a / 2
        else a := 3   a + 1
        output (a)
    end
```

49. Which of the following correctly represents the algorithm described above?

(A) $a_n = \begin{cases} \frac{1}{2}a_{n-1} \text{ for } a_{n-1} \text{ odd} \\ 3a_{n-1}+1 \text{ for } a_{n-1} \text{ even} \end{cases}$

(B) $a_n = \begin{cases} \frac{1}{2}a_{n-1} \text{ for } a_{n-1} \text{ even} \\ 3a_{n-1}+1 \text{ for } a_{n-1} \text{ odd} \end{cases}$

(C) $a_n = \begin{cases} \frac{1}{2}a_{n-1} \text{ for } a_{n-1} \text{ even} \\ \frac{1}{2}\left(3a_{n-1}+1\right) \text{ for } a_{n-1} \text{ odd} \end{cases}$

(D) $a_n = \begin{cases} \frac{1}{2}a_{n-1} \text{ for } a_n \text{ even} \\ 3a_{n-1}+1 \text{ for } a_n \text{ odd} \end{cases}$

(E) $a_n = \begin{cases} \frac{1}{2}a_{n-1} \text{ for } a_n \text{ odd} \\ 3a_{n-1}+1 \text{ for } a_n \text{ even} \end{cases}$

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50. What is the sequence of outputs when the input value is 17?

(A) 17, 52, 28, 14, 7, 20, 10, 5, 16, 8, 4, 2, 1
(B) 52, 28, 14, 7, 20, 10, 5, 16, 8, 4, 2, 1
(C) 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
(D) 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1
(E) The sequence does not end.
