Topology Qual Workshop Day 3: Separation Axioms Definitions

- If a space X has a countable basis for its topology, then X is said to be *second countable.*
- A subset A of a space X is *dense* in X if $\overline{A} = X$.
- A space having a countable dense subset is *separable.*
- X is T_0 if for any two distinct points $a, b \in X$, there is an open set U containing one of a or b but not both.
- X is T_1 if for any two distinct points $a, b \in X$, there are open sets U, V in X with $a \in U$, $b \notin U$ and $b \in V$, $a \notin V$.
- X is T_2 (Hausdorff) if for any two distinct points $a, b \in X$, there are disjoint open sets U, V in X with $a \in U$ and $b \in V$.
- X is T_3 (Regular) if X is T_1 and for any point$a \in X$ and closed set B in X with $a \notin B$, there are disjoint open sets U, V in X with $a \in U$ and $B \subseteq V$.
- X is T_4 (Normal) if X is T_1 and for any two disjoint closed sets A, B in X, there are disjoint open sets U, V in X with $A \subseteq U$ and $B \subseteq V$.

Useful facts that you should prove:

- X is regular iff X is T_1 and for all $x \in X$ and all open sets U containing x, there is an open set V of x such that $\overline{V} \subseteq U$.
- X is normal iff X is T_1 and for all closed sets $A \subseteq X$ and all open sets U containing A, there is an open set V containing A such that $\overline{V} \subseteq U$.

Useful Examples:

- $\left(\mathbb{R}, \tau_{\text{Finite complement}}\right)$ is T_1 but not T_2.
- $X = \{a, b\}$, $\tau = \{\emptyset, \{a\}, X\}$ is T_0 but not T_1.
- For any space X, with power set topology (Discrete topology) is T_4.