Topology Qualifying Exam Workshop

May 2020

Worksheet 3B

Covering spaces: Covering map, Lifting theorems; covering space group action; universal covering, Cayley complex; Galois Correspondence Theorem, deck transformation, normal covering; applications to group theory.

Part 1 - 5-10 minutes

Warm-up:

1. Find the universal covers of these spaces:
   (a) $X = S_1$.
   (b) $Y = S_1 \vee S_1$ (two circles glued together along a point).
2. What is a deck transformation? Describe the deck transformations from one of the universal covers above.
3. State the Galois Correspondence Theorem for a path-connected, locally path-connected, and semilocally simply connected space X.

Part 2 - 1 hour and 40 minutes

Try these problems:

1. (June 2014) Use covering space theory to prove that if H is a finite index subgroup of a finitely presented group, then H is finitely presented.
2. (May 2018) Let F_n denote the free group on n letters. Use covering space theory to show that for all n, there exists a subgroup H_n of F_2 such that $H_n \cong F_n$. For your choices of H_n constructed, determine if each H_n is normal in F_2.
3. (January 2019) Suppose $p : \tilde{X} \to X$ is a covering space and X is Hausdorff. Prove that $\tilde{X}$ is Hausdorff.
4. (May 2019) Let $p : \tilde{X} \to X$ and $q : \tilde{Y} \to Y$ be covering maps such that both $\tilde{X}$ and $\tilde{Y}$ are connected and simply connected. Show that for every map $f : X \to Y$ there is a map $\tilde{f} : \tilde{X} \to \tilde{Y}$ such that $f \circ p = q \circ \tilde{f}$.
5. (January 2020) Suppose Y is Hausdorff, path-connected, locally path-connected, and semilocally simply-connected, and that its universal cover $\tilde{Y}$ is compact. Prove that $\pi_1(Y)$ is finite.

Part 3 - 5-10 minutes

Wrap-up discussion with everyone:

1. Which of these problems did you find easy? Hard?
2. Which topics in the theme do you need to review more?