Basic Examination

The following is the syllabus for the Basic Examination *(for M.A. and Ph.D.)*

Fundamentals of analysis:

1. One-variable calculus foundations: completeness of the real numbers, sequences, series, limits, continuity, including epsilon-delta arguments, maxima and minima, uniform continuity, definition of the derivative, the mean value theorem, Taylor expansion with remainder, Riemann integral, mean value theorem for integrals, fundamental theorem of calculus, sequences and series of functions, uniform convergence and integration, differentiation under the integral sign, contraction maps, fixed point theory, with applications to Newton's method and solutions of non-linear equations, numerical integration with error estimation.
2. Metric space topology and analysis, primarily in R^n: open and closed sets, completeness, convergence of sequences of numbers and functions, closure, compactness, connectedness, uniform continuity, equicontinuity, countability and uncountability (e.g. of the reals), spaces of functions. Basic arguments and theorems of undergraduate analysis using these concepts, including the Bolzano-Weierstrass, the Stone-Weierstrass, and the Arzela-Ascoli theorems.
3. Multivariable calculus: definition of differentiability in several variables (approximating linear transformation), partial derivatives, chain rule, Taylor expansion in several variables, inverse and implicit function theorems, equality of mixed partials, multivariable integration, change of variables formula.

Linear algebra:
Vector spaces, subspaces, basis and dimension, linear transformations and matrices, rank and nullity, change of basis and similarity of matrices, inner product spaces, orthogonality and, orthonormality, Gram-Schmidt process, adjoints of linear transformations and dual spaces, quadratic forms and symmetric matrices, orthogonal and unitary matrices, diagonalization of hermitian and symmetric matrices, eigenvectors and eigenvalues, and their computation, exponentiation of matrices and application to differential equations, least squares problems, trace, determinant, canonical forms. Systems of linear equations: solvability criteria, Gaussian elimination, row-reduced form, LU decomposition.

Suggested References:
For Analysis and Multivariate Calculus:

1. T. Tao, *Analysis I and II*
2. T. Gamelin and R. Greene, *Introduction to Topology*, Chapter 1
3. C. H. Edwards, *Advanced Calculus of Several Variables*, Chapters I-III

Each of the following texts also covers much of the analytic material on the syllabus of the Basic Exam:

4. T. Apostol, *Mathematical Analysis*
5. M. Rosenlicht, *Introduction to Analysis*
6. W. Rudin, *Principles of Mathematical Analysis*

For Linear Algebra:

1. *Linear Algebra and Lecture Notes,* by Peter Petersen (available at http://www.math.ucla.edu/~petersen/)
2. Serge Lang, *Linear Algebra*
3. K. Hoffman, *Linear Algebra*
4. M. Marcus and H. Minc, *Introduction to Linear Algebra*
5. *Schaum's Outline of 3000 Solved Problems of Linear Algebra* (a good source of exercises).

The following texts may be useful for Basic Exam preparation for students with an interest or background in Applied Mathematics:

1. D. Serre, *Matrices: Theory and Applications*
2. A. Ralston and P. Rabinowitz, "A First Course in Numerical Analysis", 2nd edition, Chapters 9 & 10
3. K. Atkinson, "An Introduction to Numerical Analysis", 2nd edition,Chapters 7, 8 & 9.
4. M. Marcus and H. Minc, *Introduction to Linear Algebra*
5. *Schaum's Outline of 3000 Solved Problems of Linear Algebra* (a good source of exercises).