CHAPTER 1 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Find the sum of the roots of the equation $\sqrt{x-1}+\sqrt{2x-1}=x$.

(A) 1 (B) 2 (C) 4 (D) 5 (E) 6

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2. Determine the set of positive values of x that satisfy the following inequality:

$$\frac{1}{x}-\frac{1}{x-1}>\frac{1}{x-2}$$

(A) $(0,1)\cup(\sqrt{2},2)$ (B) $(0,\frac{1}{2})\cup(1,2)$ (C) $(\frac{1}{2},1)\cup(\sqrt{2},2\sqrt{2})$

(D) $(0,\sqrt{2})\cup(\frac{3}{2},2)$ (E) $(1,\sqrt{2})\cup(2,2\sqrt{2})$

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3. Solve for x: $|x+1|-|x|+2|x-1|=2x-1$

(A) $x=-\frac{1}{2},1$ (B) $x=-\frac{1}{2},2$ (C) $x=1,2$ (D) $x=-\frac{1}{2},1,2$ (E) $x\geq 1$

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4. Let f be a function such that $f(n+1)=1-[f(n)]^2$ for all nonnegative integers n. Which of the following correctly expresses $f(n+2)$ in terms of $f(n)$?

(A) $2[f(n)]^2$ (B) $2f(n)-2[f(n)]^2$ (C) $2f(n)+2[f(n)]^2$

(D) $2[f(n)]^2-[f(n)]^4$ (E) $2[f(n)]^2+[f(n)]^4$

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5. Let f be a real-valued function whose inverse is given by the equation:

$$f^{-1}(x)=x(1+x^2)+(1-x^2)$$

What's the value of $f(f^{-1}(f(2)))$?

(A) –2 (B) –1 (C) 1 (D) 2 (E) 7

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6. Let f, g, and h be real-valued functions defined for all positive x such that:

$$(f \circ g)(x) = (g \circ h)(x)$$

If $f(x) = x + 1$ and $g(x) = \sqrt{x}$, what is $h(x)$?

(A) $x^2 - 1$ (B) $\sqrt{x-1}$ (C) $\sqrt{x}+2$ (D) $x\sqrt{x}+1$ (E) $1+(\sqrt{x}+2)\sqrt{x}$

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7. What's the equation of all points in the xy-plane that are equidistant from the points $(-1, 4)$ and $(5, -2)$?

(A) $2x - y = 3$ (B) $x - y = 1$ (C) $x + y = 3$

(D) $y = x^2 - 4x + 1$ (E) $(x - 2)^2 + (y - 1)^2 = 18$

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8. Which of the following best describes the graph of the equation $x^2 + y^2 - 2x + 4y + 5 = 0$ in the xy-plane?

(A) circle (B) parabola (C) ellipse (D) line (E) point

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9. Let C be the curve in the xy-plane described by the equation $x^2 + 4y^2 = 16$. If every point (x, y) on C is replaced by the point $(\frac{1}{2}x, y)$, what is the area enclosed by the resulting curve?

(A) 8 (B) 4π (C) 16 (D) 8π (E) 16π

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10. Every point on the parabola $y = \sqrt{2x-1}$ is equidistant from the y-axis and which of the following points?

(A) $(\frac{1}{2}, 0)$ (B) $(1, 0)$ (C) $(\frac{3}{2}, 0)$ (D) $(2, 0)$ (E) $(\frac{5}{2}, 0)$

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11. One of the foci of the hyperbola $y^2 = \left(\frac{x}{a}\right)^2 + 1$ is the point $(0, \sqrt{2})$. Find a.

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{2}$ (D) 1 (E) $\sqrt{3}$

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12. Which one of the following polynomials $p(x)$ has the property that $\sqrt{3} - \sqrt{2}$ is a root of the equation $p(x) = 0$?

(A) $2x^2 + 6x + 3$ (B) $x^3 - 2x + 6$ (C) $x^4 + 2x^2 - 3$ (D) $x^4 - 10x^2 + 1$ (E) $x^4 - 5x^2 + 6$

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13. When the polynomial $p(x)$ is divided by $x - 1$, it leaves a remainder of 1, and when $p(x)$ is divided by $x + 1$, it leaves a remainder of -1. Find the remainder when $p(x)$ is divided by $x^2 - 1$.

(A) -1 (B) 0 (C) x (D) $-x$ (E) $2x$

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14. Given that $p(x)$ is a real polynomial of degree ≤ 4 such that one can find five distinct solutions to the equation $p(x) = 5$, what is the value of $p(5)$?

(A) 0 (B) 1 (C) 4 (D) 5
(E) Cannot be determined from the information given

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15. If the roots of the equation $x^2 + Bx + 1 = 0$ are the squares of the roots of the equation $x^2 + bx + 1 = 0$, which of the following expresses B in terms of b?

(A) $2 - b^2$ (B) $1 - b^2$ (C) $b^2 - 1$ (D) b^2 (E) $b^2 - 2$

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16. Find the largest value of b such that $1 + bi$ satisfies the equation

$$x^3 - 3x^2 + 6x - 4 = 0$$

given that *every* root of this equation has the form $1 + bi$ (where b is real).

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 (E) 3

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17. If a and x are positive numbers and $A = a^2$, express the following in its simplest form in terms of x:

$$a^{(\log_a x) + (\log_A x)}$$

(A) $2x$ (B) x^2 (C) $\sqrt{x}$ (D) x^3 (E) $x\sqrt{x}$

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18. What are the roots of the following equation?

$$(\log x)^2 = 2 \log x$$

(A) $1, e^{-2}$ (B) $1, \sqrt{e}$ (C) $1, e^2$ (D) all $x > 0$ (E) all real x

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19. The hyperbolic sine function, denoted sinh, is defined by the equation:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

Find a formula for $\sinh^{-1} x$.

(A) $\log(1-\sqrt{x^2+1})$ (B) $\log(1+\sqrt{x^2+1})$ (C) $\log(x-\sqrt{x^2+1})$
(D) $\log(x-\sqrt{x^2+1})$ (E) $\log(\sqrt{x^2+1}-x)$

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20. The hyperbolic cosine function, denoted cosh, is defined by the equation:

$$\frac{e^x + e^{-x}}{2}$$

If the hyperbolic tangent function, tanh, is defined by

$$\tanh x = \frac{\sinh x}{\cosh x}$$

find a formula for $\tanh^{-1} x$.

(A) $\frac{1}{2}\log\frac{x-1}{x+1}$ (B) $\log\frac{\frac{1}{2}x-1}{\frac{1}{2}x+1}$ (C) $\log\frac{1+\frac{1}{2}x}{1-\frac{1}{2}x}$ (D) $\frac{1}{2}\log\frac{x+1}{x-1}$ (E) $\frac{1}{2}\log\frac{1+x}{1-x}$

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21. Let x be the real number such that $\sin(\sin x) = \frac{1}{2}$ and $2 < x < 3$. What's the value of $\cos(-\sin x)$?

(A) $-\sqrt{1-\left(\frac{\pi}{6}\right)^2}$ (B) $\sqrt{1-\left(\frac{\pi}{3}\right)^2}$ (C) $\sqrt{1-\left(\frac{\pi}{6}\right)^2}$ (D) $-\frac{\sqrt{3}}{2}$ (E) $\frac{\sqrt{3}}{2}$

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22. Which one of the following is in the domain of the function $f(x) = \log(\sin x)$? (You may use the fact that 1111 is just slightly greater than $353.64 \times \pi$.)

(A) 11 (B) 111 (C) 1111 (D) 11,111 (E) None of these

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23. Simplify $\tan(2 \arcsin\frac{1}{3})$.

(A) $\frac{2\sqrt{2}}{9}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4\sqrt{2}}{7}$ (E) $\frac{6}{5}$

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24. Simplify $\sqrt{\csc^2\left(\operatorname{arccot}\frac{\pi}{4}\right)-1}$.

(A) -1 (B) 1 (C) $\frac{\pi^2}{16}$ (D) $\frac{\pi}{4}$ (E) $\frac{\sqrt{\pi}}{2}$

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25. Determine the exact value of the sum arctan 1 + arctan 2 + arctan 3.

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) $\frac{\pi}{4}-1$ (E) $\frac{\pi}{2}-1$
