CHAPTER 4 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. Let $y = f(x)$ be the solution of the equation

$$\frac{dy}{dx} = \frac{x^2}{x^2+1}$$

such that $y = 0$ when $x = 0$. What is the value of $f(1)$?

(A) $1 - \log 2$ (B) $1 + \log 2$ (C) 1 (D) $\log 2$ (E) $\frac{1}{4}(4 - \pi)$

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2. A population of bacteria grows at a rate proportional to the number present. After two hours, the population has tripled. After two more hours elapse, the population will have increased by a factor of k. What is the value of k?

(A) 6 (B) 8 (C) 9 (D) 27 (E) 81

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3. Every curve in a certain family, $y = f(x, c)$, has the following property: the area of the region in the first quadrant bounded above by the curve from $(0, 0)$ to (x, y) and bounded below by the x-axis is $\frac{1}{3}$ the area of the rectangle with opposite vertices at $(0, 0)$ and (x, y). Find $f(x, c)$.

(A) cx^3 (B) $cx^3 + x$ (C) $cx^3 - x$ (D) cx^2 (E) $c\sqrt{x}$

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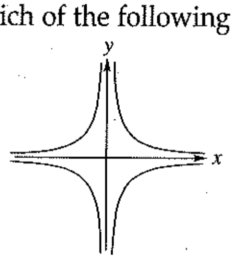
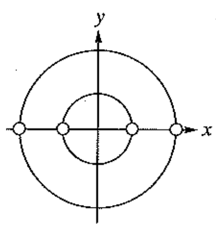
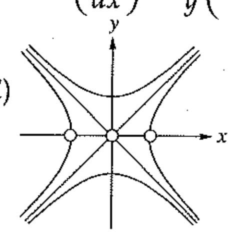
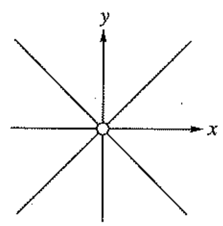
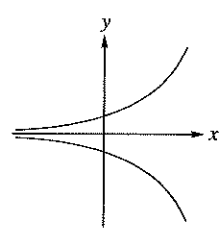
4. Which of the following depicts integral curves of the differential equation $\left(\frac{dy}{dx}\right)^2 = \frac{x}{y}\left(2\frac{dy}{dx} - \frac{x}{y}\right)$?

(A)


(B)


(C)


(D)


(E)


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5. If a is a positive constant, let $y = f(x)$ be the solution of the equation

$$y''' - ay'' + a^2y' - a^3y = 0$$

such that $f(0) = 1$, $f'(0) = 0$, and $f''(0) = a^2$. How many positive values of x satisfy the equation $f(x) = 0$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

6. Let g: $\mathbf{R} \to \mathbf{R}$ be a differentiable and integrable function. The integral curve of the differential equation

$$[y + g(x)]\, dx + [x - g(y)]\, dy = 0$$

that passes through the point (1, 1) must also pass through which of the following points?

(A) (0, 0) (B) $(2, \frac{1}{2})$ (C) $(\frac{1}{2}, 2)$ (D) (–1, –1) (E) (0, 1)

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7. Let $y = f(x)$ be the solution of the equation

$$\frac{dy}{dx} + \frac{y}{x} = \sin x$$

such that $f(\pi) = 1$. What is the value of $f(\frac{1}{2}\pi)$?

(A) $\frac{2}{\pi} - 1$ (B) $\frac{2}{\pi}$ (C) $\frac{2}{\pi} + 1$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{2} + 1$

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8. Let $y = f(x)$ be the solution of the equation

$$\frac{d^4y}{dx^4} = \frac{d^2y}{dx^2}$$

such that $f(0) = f'(0) = f''(0) = 0$ and $f'''(0) = -1$. What is $f(x)$?

(A) $x - \cosh x$ (B) $x - \sinh x$ (C) $x + \cosh x$ (D) $x + \sinh x$ (E) $\cosh x + \sinh x$

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9. What is the general solution of the differential equation

$$2\frac{d^3x}{dt^3} + 7\frac{d^2x}{dt^2} + 3\frac{dx}{dt} = 6?$$

(A) $x = 2t + c_1e^t + c_2e^{-t/2} + c_3e^{-3t}$ (B) $x = 2 + c_1e^t + c_2e^{-t/2} + c_3e^{-3t}$ (C) $x = t^2 + c_1 + c_2e^{-t/3} + c_3e^{-2t}$

(D) $x = 2t + c_1 + c_2e^{-t/3} + c_3e^{-2t}$ (E) $x = 2t + c_1 + c_2e^{-t/2} + c_3e^{-3t}$

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10. Given that the following differential equation has an integrating factor of the form $\mu(x, y) = x^my^n$, determine its general solution.

$$(3xy^2 - 5y)\, dx + (2x^2y - 3x)\, dy = 0$$

(A) $x^4y^2(\frac{1}{2}xy - 1) = c$ (B) $x^4y^2(xy - 1) = c$ (C) $x^4y^2(2xy - 1) = c$

(D) $x^5y^3(\frac{1}{2}xy - 1) = c$ (E) $x^5y^3(2xy - 1) = c$

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11. At every point (x, y) on a curve in the xy-plane, the slope is equal to:

$$\frac{1-2xy}{x^2+3y^2+1}$$

What is the equation of this curve, given that it passes through the point (1, 1)?

(A) $\frac{1}{3}x^3 + 3xy^2 + x + y - xy^2 = \frac{13}{3}$
(B) $xy^2 + y^3 + x - y = 2$
(C) $\frac{1}{3}x^3 + 3xy^2 - x + y + xy^2 = \frac{13}{3}$
(D) $x^2y + y^3 - x + y = 2$
(E) $x^2y^2 + xy^3 + x - y = 1$

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12. Find the general solution of the differential equation:

$$\frac{dy}{dx} = \frac{x+y}{x}$$

(A) $e^{y/x} = cx$ (B) $e^{y/x} = cy$ (C) $e^{x/y} = cx$ (D) $e^{x/y} = cy$ (E) $e^{-x/y} = cx$

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13. Consider the family F of circles in the xy-plane, $(x - c)^2 + y^2 = c^2$, that are tangent to the y-axis at the origin. Which of the following gives the differential equation that is satisfied by the family of curves orthogonal to F?

(A) $y' = \frac{x}{x-y}$ (B) $y' = \frac{x}{y-x}$ (C) $y' = \frac{xy}{x-y}$ (D) $y' = \frac{2xy}{x^2-y^2}$ (E) $y' = \frac{2xy}{y^2-x^2}$

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14. Let $g(x, y)$ be the function defined for all x and all nonzero y such that the differential equation

$$(\sin xy)\, dx + g(x, y)\, dy = 0$$

is exact and $g(0, y) = 0$ for all $y \neq 0$. What is $g(x, 1)$?

(A) $\sin x + \cos x - 1$
(B) $x \sin x + \cos x - 1$
(C) $x \sin x - \cos x + 1$
(D) $x \sin x + \cos x$
(E) $\sin x - x \cos x + 1$

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15. If $w = f(x, y)$ is a solution of the partial differential equation

$$2\frac{\partial w}{\partial x} - 3\frac{\partial w}{\partial y} = 0$$

then w could equal

(A) $(2x - 3y)^6$
(B) $\sin\left[\log(3x-2y)\right]$
(C) $e^{\arctan(3x+2y)}$
(D) $\left[\arccos(2y-3x)\right]^2$
(E) $\sqrt{2x+3y}$
