CHAPTER 6 REVIEW QUESTIONS

Complete the following review questions using the techniques outlined in this chapter. Then, see Chapter 8 for answers and explanations.

1. There is only one integer, x, between 100 and 200 such that integer pair (x, y) satisfies the equation $42x + 55y = 1$. What's the value of x in this integer pair?

   (A) 127 (B) 148 (C) 158 (D) 167 (E) 183

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2. Let L be the least common multiple of 1001 and 10101. What's the sum of the digits of L? (All numbers are written in their usual decimal representation.)

   (A) 6 (B) 11 (C) 17 (D) 22 (E) 33

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3. Let x_1 and x_2 be the two smallest positive integers for which the following statement is true: "$85x - 12$ is a multiple of 19." Then $x_1 + x_2 =$

   (A) 19 (B) 27 (C) 31 (D) 38 (E) 47

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4. If x, y, and z are positive integers such that $4x - 5y + 2z$ is divisible by 13, then which one of the following must also be divisible by 13?

   (A) $x + 13y - z$ (B) $6x - 10y - z$ (C) $x - y - 2z$ (D) $-7x + 12y + 3z$ (E) $-5x + 3y - 4z$

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5. When expressed in its usual decimal notation, the number 100! (that is, 100 factorial) ends in how many consecutive zeros?

   (A) 20 (B) 24 (C) 30 (D) 32 (E) 50

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6. How many generators does the group $(\mathbf{Z}_{24}, +)$ have?

   (A) 2 (B) 6 (C) 8 (D) 10 (E) 12

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7. Which one of the following groups is cyclic?

   (A) $\mathbf{Z}_2 \times \mathbf{Z}_4$ (B) $\mathbf{Z}_2 \times \mathbf{Z}_6$ (C) $\mathbf{Z}_3 \times \mathbf{Z}_4$ (D) $\mathbf{Z}_3 \times \mathbf{Z}_6$ (E) $\mathbf{Z}_4 \times \mathbf{Z}_6$

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8. If G is a group of order 12, then G must have a subgroup of all of the following orders EXCEPT

   (A) 2 (B) 3 (C) 4 (D) 6 (E) 12

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9. How many subgroups does the group $\mathbf{Z}_3 \oplus \mathbf{Z}_{16}$ have?

(A) 6 (B) 10 (C) 12 (D) 20 (E) 24

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10. If $S = \{a \in \mathbf{R}^+ : a \neq 1\}$, with the binary operation $\bullet$ defined by the equation $a \bullet b = a^{\log b}$ (where $\log b = \log_e b$), then $(S, \bullet)$ is a group. What is the inverse of $a \in S$?

(A) $\dfrac{1}{e \log a}$ (B) $\dfrac{e}{\log a}$ (C) $e^{-\log a}$ (D) $e^{\log(1/a)}$ (E) $e^{1/\log a}$

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11. Which of the following are subgroups of GL(2, **R**), the group of invertible 2 by 2 matrices (with real entries) under matrix multiplication?

I. $T = \{A \in GL(2,\mathbf{R}): \det A = 2\}$

II. $U = \{A \in GL(2,\mathbf{R}): A \text{ is upper triangular}\}$

III. $V = \{A \in GL(2,\mathbf{R}): \text{tr}(A) = 0\}$

Note: tr(A) denotes the *trace* of A, which is the sum of the entries on the main diagonal.

(A) I and II only (B) II only (C) II and III only (D) III only (E) I and III only

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12. Let p and q be distinct primes. How many (mutually nonisomorphic) Abelian groups are there of order p^2q^4?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 16

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13. Let G be the group generated by the elements x and y and subject to the following relations: $x^2 = y^3$, $y^6 = 1$, and $x^{-1}yx = y^{-1}$. Express in simplest form the inverse of the element $z = x^{-2}yx^3y^3$.

(A) $y^{-2}x^{-1}$ (B) xy^2 (C) xy (D) yx (E) y^2x

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14. Let H be the set of all group homomorphisms $\phi: \mathbf{Z}_3 \to \mathbf{Z}_6$. How many functions does H contain?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

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15. Let G be a group of order 9, and let e denote the identity of G. Which one of the following statements about G CANNOT be true?

(A) There exists an element x in G such that $x \neq e$ and $x^{-1} = x$.

(B) There exists an element x in G such that $x \neq e$ and $x^2 = x^5$.

(C) There exists an element x in G such that $\langle x \rangle$ has order 3.

(D) G is cyclic.

(E) G is Abelian.

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16. Let R be a ring; an element x in R is said to be idempotent if $x^2 = x$. How many idempotent elements does the ring $\mathbf{Z}_{20}$ contain?

(A) 2 (B) 4 (C) 5 (D) 8 (E) 10

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17. Which of the following rings are integral domains?

I. $\mathbf{Z} \oplus \mathbf{Z}$
II. $\mathbf{Z}_p$, where p is a prime
III. $\mathbf{Z}_{p^2}$, where p is a prime

(A) I and II only (B) II only (C) II and III only (D) III only (E) I and III only

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18. Which one of the following rings does NOT have the same number of units as the other four?

(A) $\mathbf{Z} \oplus \mathbf{Z}$ (B) $\mathbf{Z} \oplus \mathbf{Z}_3$ (C) $\mathbf{Z} \oplus \mathbf{Z}_5$ (D) $\mathbf{Z} \oplus \mathbf{Z}_6$ (E) $\mathbf{Z}_3 \oplus \mathbf{Z}_3$

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19. How many elements x in the field $\mathbf{Z}_{11}$ satisfy the equation $x^{12} - x^{10} = 2$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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20. Which of the following are subfields of $\mathbf{C}$?

I. $K_1 = \left\{a + b\sqrt{\tfrac{2}{3}} : a,\ b \in \mathbf{Q}\right\}$

II. $K_2 = \left\{a + b\sqrt{2} : a,\ b \in \mathbf{Q} \text{ and } ab < \sqrt{2}\right\}$

III. $K_3 = \left\{a + bi : a,\ b \in \mathbf{Z} \text{ and } i = \sqrt{-1}\right\}$

(A) I only (B) I and II only (C) III only
(D) I and III only (E) None of the K_i are subfields of $\mathbf{C}$.
